A common element of time-series data in fields such as geophysics, hydrology, and astronomy is seasonal signal. This variation indubitably obscures the trends that exist in the time-series. In cases with seasonal signals, the slope estimates can have low bias as long as the data span is long enough to outweigh the effects of the variation of the seasonal signal. The traditional method of calculating slopes of time-series is least-squares regression. Bleviss and Lavallée [2002] conclude that least-squares slope estimates that account for seasonal signal produce reasonable biases for data spans above 2.5 years long. They add that least squares slope estimates not accounting for seasonal signal produce acceptable biases for data spans beyond 4.5 years. The Theil-Sen method is a nonparametric approach to time-series regression that does not depend on the bold assumptions of parametric methods like least-squares. This method is additionally robust to outliers, whereas least-squares can be heavily influenced by the presence of outliers.

Methods

Theil-Sen Estimator

There are \( \binom{n}{2} \) possible pairs of points in a time-series of \( n \) data points and each pair has a characteristic, but not always unique, slope. Theil [1950] proposes using the median of these slopes as an estimator for the regression coefficient. The seasonal estimator matches only data points within a range of exactly an integer number of years plus/minus a time interval, \( dt \). For time-series based on the GPS position data of interest, this is an estimate of tectonic plate velocity. As this estimator is insensitive to outliers and does not rely on the strong assumptions of the least-squares estimator, it may produce smaller biases compared to the default least-squares approach.

Time-Series Simulations

(1) Generate set of 250 artificial time-series for each combination of: true velocity (mm/yr), white noise amplitude, seasonal signal amplitude, time length (yr); and starting phase (rad) for seasonal signal, and \( dt \) (days) for Theil-Sen

(2) Calculate seasonal and nonseasonal least-squares velocity estimates and a seasonal Theil-Sen velocity estimate

(3) Average velocity estimates over the 250 for each type of estimator

(4) Calculate the absolute bias for each method from the known true velocity

Application to Real Data

SUNS (Figure 2) is chosen as a reliable MAGNET station consistently \( \geq 2.5 \) years producing position measurements from which a seasonal least-squares velocity estimate can be used as truth.

(1) Calculate Theil-Sen velocity estimates for varying time lengths (spanning from 1.1 years to 2.5 years) and chosen \( dt \) (spanning from 0 to 30 days) over varying sections in the time-series

(2) Calculate the absolute bias for each estimate from the velocity from the seasonal least-squares estimate over the entire available data (\( \geq 2.5 \) years), which is considered the true velocity

Results

Figure 4. Theil-Sen Seasonal increment, \( dt \) (in days) versus absolute velocity bias (in mm/yr) using the [A] non-seasonal least-squares estimator, [B] seasonal least-squares estimator, [C] seasonal Theil-Sen estimator with \( dt = 5 \) days; circle size is proportional to amplitude of seasonal signal large: 10, medium: 1, small: 1.1

Figure 5. Data span time length (in years) versus absolute velocity bias (in mm/yr) using the [A] non-seasonal least-squares estimator, [B] seasonal least-squares estimator, [C] seasonal Theil-Sen estimator with \( dt = 5 \) days; circle size is proportional to amplitude of seasonal signal large: 10, medium: 1, small: 0.1

Discussion

While this study is rigorous in realizations of white noise, seasonal signal, phase, and relevant data span lengths, it lacks considerations of other phenomena in GPS time-series. Such phenomena include flicker noise, additional colored noise, and steps. Therefore, the current findings that the seasonal Theil-Sen estimator is, at best, equally as effective as the seasonal least-squares estimator may be premature. The additional garnered information about the seasonal Theil-Sen estimator will provide a foundation for future studies comparing these estimators. These future studies might seek to increase the rigor in simulating time-series with more detailed noise. The comparison of these estimators is also important to view with more rigor in the context of real data from stations like SUNS.

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